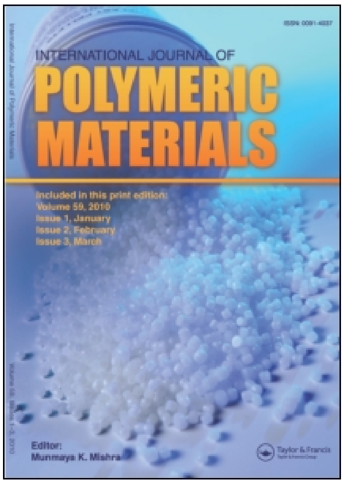

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Broken Section Method for Analyzing Non-Symmetrical Calendering of Polymeric Materials

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It is proposed in this paper that non-symmetrical calendering can theoretically be analyzed easily by the broken section method. We assume that the flow is steady, isothermal and laminar and the polymeric materials behave as a power law fluid with flow index n. In the direction of calendering, polymeric materials between two rolls are broken into many sections, each of unit length. The materials in each section having different heights related to the clearance between two rolls behave under combined drag and pressure flow of power law fluids between parallel plates. The pressure distribution in the direction of calendering can be solved in four cases where the relationships between drag and pressure flows are varied, and, in addition, the position of maximum pressure can be determined. The theoretical results on the velocity profiles of polymeric materials in calendering can be obtained with better accuracy in the narrow nip region.

I. INTRODUCTION

The broken section method has been applied successfully to the analysis of various boundary value problems in polymer processing,[1] especially on

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extrusion die design problem.[2-5] Non-symmetrical calendering of polymeric materials has been solved by bi-polar[6] coordinates. However the analytical results solved rigorously by bi-polar coordinates are very complicated. An easy and convenient approach to analysis on non-symmetrical calendering of polymeric materials by the broken section method is reported in this paper.

II. FUNDAMENTAL

Because two-dimensional flow is assumed in this analysis, the Cartesian coordinate for non-symmetrical calendering by a pair of driven rolls with different diameters ($R_1 \neq R_2$) and different rotating peripheral velocities ($U_1 \neq U_2$), is employed, as shown in Figure II–1. It seems to be reasonable that the x-axis is the neutral axis decided by the bi-polar[6] coordinate system, while the y-axis is located in the nip section.

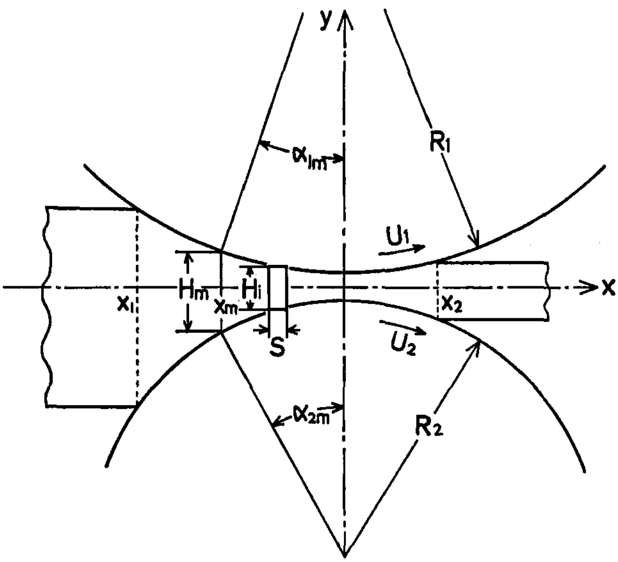

FIGURE II–1 Notation for analysis of flow in non-symmetrical calendering.

It is assumed that flow is steady, isothermal and laminar and that the power law is applicable to polymeric materials. The power law is written in the form[7]

$$\tau = \eta \dot{\gamma} \tag{II–1}$$

$$\eta = \eta^0 \left(\frac{\dot{\gamma}}{\dot{\gamma}^0}\right)^{(n-1)} \tag{II–2}$$

where η is the viscosity at shear rate $\dot{\gamma}$. The flow index of the fluid is n and η^0 is the viscosity at the standard shear rate $\dot{\gamma}^0$.

The total contacting length of polymeric materials with the rolls in the x-axis is X.

$$X = |x_1| + |x_2| \tag{II–3}$$

where

x_1: the distance between entrance and nip
x_2: the distance between exit and nip.

It is assumed that the nip is small in comparison to R_1 or R_2 and so the flow velocity v_y in the y-direction can be neglected. Furthermore, it is assumed that the variation in the flow velocity v_x in the x-direction is much less in the x-direction than in the y-direction. Hence the derivatives of v_x with respect to x can be ignored, leaving only derivatives with respect to y. The additional simplified assumption, that the hydrostatic pressure varies only in the x-direction, is made. Thus it has been analyzed theoretically and experimentally that the pressure profile of calendering in the x-direction has a maximum at $x = x_m$ between entrance and nip. Note that the pressure gradients (dp/dx) in the x-direction have the positive and negative values at the regions for $x_1 < x < x_m$ and $x_m < x < x_2$ respectively.

III. BROKEN SECTION METHOD

For the purpose of the analysis by the broken section method, the total contacting length X is split into N sections, each of width W and length S, where

$$S = X/N. \tag{III–1}$$

As shown in Figure III–1, the height of the ith section is H_i where it is expressed in terms of R_1, R_2 and x

$$H_i = |H_{1i}| + |H_{2i}| \tag{III–2}$$

where

H_{1i}: the upper height of the ith section from the x-axis
H_{2i}: the lower height of the ith section from the x-axis.

The pressure at the start of the ith section is p_{i-1} and at the end is p_i. Hence $(p_i - p_{i-1})/S$ is the pressure gradient through the ith section. Thus, the x-component of the momentum equation of the ith section in calendering of a power-law fluid reduces to

$$\frac{p_i - p_{i-1}}{S} = \frac{d}{dy}\left\{\frac{\eta^0}{(\dot{\gamma}^0)^{(n-1)}}\left|\frac{dv_{xi}}{dy}\right|^{(n-1)}\left(\frac{dv_{xi}}{dy}\right)\right\} \tag{III–3}$$

Under the assumption of no slip, the upper and lower surfaces of the ith section are dragged to the x-direction by a pair of driven rolls. Hence the boundary conditions are as follows.

$$v_{xi}(y = +H_{1i}) = U_{1i} \quad \text{(III-4)}$$

$$v_{xi}(y = -H_{2i}) = U_{2i} \quad \text{(III-5)}$$

where

U_{1i}: x-component of the peripheral velocity U_1 of upper roll at the part contacted with the ith section.

U_{2i}: x-component of the peripheral velocity U_2 of lower roll at the part contacted with the ith section.

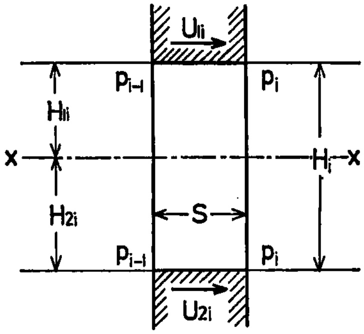

FIGURE III–1 The ith section of the polymeric materials being calendered and showing principal dimensions and flows.

By defining the following dimensionless variables

$$\bar{y} = y/H_i \quad \text{(III-6)}$$

$$\phi_i = \frac{v_{xi}}{\Gamma_i U_{1i}} \quad \text{(III-7)}$$

$$\Gamma_i = \frac{\dot{\gamma}^0 H_i}{U_{1i}}\left\{\frac{H_i}{\eta^0\dot{\gamma}^0}\left|\frac{p_i - p_{i-1}}{S}\right|\right\}^{1/n} \quad \text{(III-8)}$$

one obtains the following differential equation[8] for the flow of the ith section:

$$\frac{d}{d\bar{y}}\left\{\left|\frac{d\phi_i}{d\bar{y}}\right|^{(n-1)}\frac{d\phi_i}{d\bar{y}}\right\} = \pm 1 \quad \text{(III-9)}$$

where the sign on the right hand side is positive when $(p_i - p_{i-1})/S > 0$ and negative when $(p_i - p_{i-1})/S < 0$. Depending upon the mutual relationship

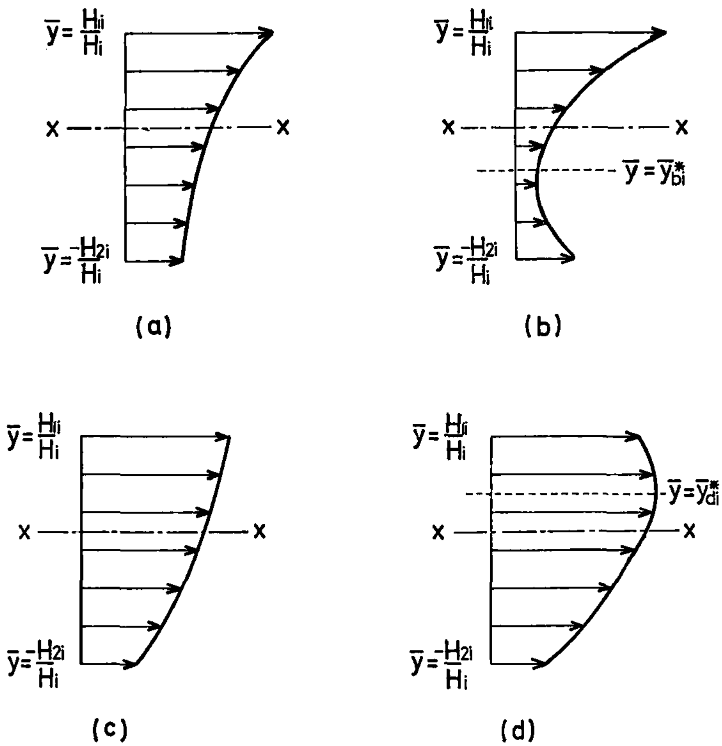

FIGURE III–2 Types of velocity profiles in flow of polymeric materials between a pair of driven rolls; when $(dp/dx) > 0$ (a), (b) and when $(dp/dx) < 0$ (c), (d).

between the pressure flow by $(p_i - p_{i-1})/S$ and the drag flow by U_{1i} and U_{2i}, the four[4] velocity profiles are considered, as shown in Figure III–2: (a), (b), (c) and (d). The theoretical analysis on the flow in the ith section for each case is presented in the following section to determine the pressure and the v_{xi} component as a function of the position variables x and y.

IV. ANALYSIS ON FLOW IN BROKEN SECTION

(a) Figure III-2(a), $(p_i - p_{i-1})/S > 0$.

The differential equation for (a) is as follows.

$$\frac{d}{d\bar{y}}\left\{\left(\frac{d\phi_i}{d\bar{y}}\right)^n\right\} = +1 \qquad \text{(IV-1)}$$

The analytical result is obtained.

$$\phi_i = \frac{(\bar{y}+K_{1i})^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}+K_{2i} \tag{IV-2}$$

where K_{1i} and K_{2i} are integration constants and can be determined by the following boundary conditions.

$$v_{xi}(\bar{y} = +H_{1i}/H_i) = U_{1i}, \quad \text{i.e.} \quad \phi_i(\bar{y} = +H_{1i}/H_i) = \Gamma_i^{-1}, \tag{IV-3}$$

$$v_{xi}(\bar{y} = -H_{2i}/H_i) = U_{2i}, \quad \text{i.e.} \quad \phi_i(\bar{y} = -H_{2i}/H_i) = \frac{U_{2i}}{\Gamma_i U_{1i}}. \tag{IV-4}$$

Namely

$$\frac{\left(+\frac{H_{1i}}{H_i}+K_{1i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}+K_{2i} = \Gamma_i^{-1} \tag{IV-5}$$

$$\frac{\left(-\frac{H_{2i}}{H_i}+K_{1i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}+K_{2i} = \frac{U_{2i}}{\Gamma_i U_{1i}}. \tag{IV-6}$$

Hence

$$K_{2i} = \Gamma_i^{-1}-\frac{\left(+\frac{H_{1i}}{H_i}+K_{1i}\right)}{\left(\frac{1}{n}+1\right)} \tag{IV-7}$$

K_{1i} is determined by the following equation.

$$\left(\frac{H_{1i}}{H_i}+K_{1i}\right)^{(1/n+1)}-\left(-\frac{H_{2i}}{H_i}+K_{1i}\right)^{(1/n+1)} = \left(\frac{1}{n}+1\right)\left(\Gamma_i^{-1}-\frac{U_{2i}}{\Gamma_i U_{1i}}\right) \tag{IV-8}$$

The volumetric flow rate Q can be expressed as

$$\frac{Q}{W} = \int_{-H_{2i}}^{+H_{1i}} v_{yi}\,dy = \Gamma_i U_{1i} H_i \int_{-H_{2i}/H_i}^{+H_{1i}/H_i} \phi_i\,d\bar{y}$$

$$= \Gamma_i U_{1i} H_i \left\{ \frac{\left(+\frac{H_{1i}}{H_i}+K_{1i}\right)^{(1/n+2)}-\left(-\frac{H_{2i}}{H_i}+K_{1i}\right)^{(1/n+2)}}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)} \right.$$

$$+\frac{1}{\Gamma_i}-\frac{\left(+\frac{H_{1i}}{H_i}+K_{1i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}\Bigg\} \quad \text{(IV-9)}$$

Γ_i and K_{1i} are a function of $(p_i-p_{i-1})/S$.

(b) Figure III–2 (b), $(p_i-p_{i-1})/S > 0$.

The velocity profile for (b) has an extreme value somewhere between two rolls, at $\bar{y} = \bar{y}^*_{bi}$. It is necessary, then, to write the differential equation for each region separately and account for the proper sign for the shear rate in the absolute value sign. In a similar way, Eq. (III–9) in the upper part of the velocity profile, where $d\phi_{Ui}/d\bar{y} > 0$, is as follows.

$$\frac{d}{d\bar{y}}\left\{\left(\frac{d\phi_{Ui}}{d\bar{y}}\right)^n\right\} = +1 \quad \left(+\frac{H_{1i}}{H_i} < \bar{y} < \bar{y}^*_{bi}\right) \quad \text{(IV-10)}$$

In the lower part of the velocity profile, where $d\phi_{Li}/d\bar{y} < 0$, on the other hand, Eq. (III–9) can be written as

$$-\frac{d}{d\bar{y}}\left\{\left(-\frac{d\phi_{Li}}{d\bar{y}}\right)^n\right\} = +1 \quad \left(\bar{y}^*_{bi} < \bar{y} < -\frac{H_{2i}}{H_i}\right) \quad \text{(IV-11)}$$

The boundary conditions for this case are as follows.

$$\bar{y} = +\frac{H_{1i}}{H_i}: \quad \phi_{Ui} = \Gamma_i^{-1} \quad \text{(IV-12)}$$

$$\bar{y} = \bar{y}^*_{bi}: \quad \frac{d\phi_{Ui}}{d\bar{y}} = 0 \quad \text{(IV-13)}$$

$$\bar{y} = \bar{y}^*_{bi}: \quad \frac{d\phi_{Li}}{d\bar{y}} = 0 \quad \text{(IV-14)}$$

$$\bar{y} = -\frac{H_{2i}}{H_i}: \quad \phi_{Li} = \frac{U_{2i}}{\Gamma_i U_{1i}} \quad \text{(IV-15)}$$

$$\bar{y} = \bar{y}^*_{bi}: \quad \phi_{Ui} = \phi_{Li} \quad \text{(IV-16)}$$

The solutions are

$$\phi_{Ui} = \frac{1}{\left(\frac{1}{n}+1\right)}\left\{(\bar{y}-\bar{y}^*_{bi})^{(1/n+1)}-\left(\frac{H_{1i}}{H_i}-\bar{y}^*_{bi}\right)^{(1/n+1)}\right\}+\Gamma_i^{-1} \quad \text{(IV-17)}$$

$$\phi_{Li} = \frac{1}{\left(\frac{1}{n}+1\right)}\left\{(-\bar{y}+\bar{y}^*_{bi})^{(1/n+1)}-\left(\frac{H_{2i}}{H_i}+\bar{y}^*_{bi}\right)^{(1/n+1)}\right\}+\frac{U_{2i}}{\Gamma_i U_{1i}} \quad \text{(IV-18)}$$

where the following equation for $\bar{y}^*_{bi}$ is obtained from the boundary condition (IV–16).

$$\left(\frac{1}{n}+1\right)\Gamma_i^{-1}-\left(\frac{H_{1i}}{H_i}-\bar{y}^*_{bi}\right)^{(1/n+1)} = \left(\frac{1}{n}+1\right)\frac{U_{2i}}{\Gamma_i U_{1i}}-\left(\frac{H_{2i}}{H_i}+\bar{y}^*_{bi}\right)^{(1/n+1)} \quad \text{(IV–19)}$$

The volumetric flow rate Q is obtained by integrating the velocity equation.

$$\frac{Q}{W} = \Gamma_i U_{1i} H_i \left\{\int_{-H_{2i}/H_i}^{\bar{y}^*_{bi}} \phi_{Li}\, d\bar{y} + \int_{\bar{y}^*_{bi}}^{+H_{1i}/H_i} \phi_{Ui}\, d\bar{y}\right\} \quad \text{(IV–20)}$$

$$\int_{-H_{2i}/H_i}^{\bar{y}^*_{bi}} \phi_{Li}\, d\bar{y} = \frac{1}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)}\left(\frac{H_{1i}}{H_i}+\bar{y}^*_{bi}\right)^{(1/n+2)}$$

$$-\left\{\frac{1}{\left(\frac{1}{n}+1\right)}\left(\frac{H_{1i}}{H_i}+\bar{y}^*_{bi}\right)^{(1/n+1)}-\frac{U_{2i}}{\Gamma_i U_{1i}}\right\}\left(\bar{y}^*_{bi}+\frac{H_{2i}}{H_i}\right) \quad \text{(IV–21)}$$

$$\int_{\bar{y}^*_{bi}}^{+H_{1i}/H_i} \phi_{Ui}\, d\bar{y} = \frac{1}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)}\left(\frac{H_{1i}}{H_i}-\bar{y}^*_{bi}\right)^{(1/n+2)}$$

$$-\left\{\frac{1}{\left(\frac{1}{n}+1\right)}\left(\frac{H_{1i}}{H_i}-\bar{y}^*_{bi}\right)^{(1/n+1)}-\Gamma_i^{-1}\right\}\left(\frac{H_{2i}}{H_i}-\bar{y}^*_{bi}\right) \quad \text{(IV–22)}$$

Γ_i and $\bar{y}^*_{bi}$ are a function of $(p_i-p_{i-1})/S$.

(c) Figure III–2 (c), $(p_i-p_{i-1})/S < 0$.

The differential equation for (c) is as follows.

$$\frac{d}{d\bar{y}}\left\{\left(\frac{d\phi_i}{d\bar{y}}\right)^n\right\} = -1 \quad \text{(IV–23)}$$

The analytical result can be written as

$$\phi_i = \frac{(-\bar{y}+K_{3i})^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}+K_{4i} \quad \text{(IV–24)}$$

where K_{3i} and K_{4i} are integration constants and can be determined by the following boundary conditions.

$$v_{xi}\left(\bar{y} = +\frac{H_{1i}}{H_i}\right) = U_{1i}, \quad \text{i.e.} \quad \phi_i\left(\bar{y} = +\frac{H_{1i}}{H_i}\right) = \Gamma_i^{-1} \quad \text{(IV–25)}$$

$$v_{xi}\left(\bar{y} = -\frac{H_{2i}}{H_i}\right) = U_{2i}, \quad \text{i.e.} \quad \phi_i\left(\bar{y} = -\frac{H_{2i}}{H_i}\right) = \frac{U_{2i}}{\Gamma_i U_{1i}} \quad \text{(IV–26)}$$

In the similar way to (a), K_{3i} and K_{4i} are determined by solving the following two simultaneous equations.

$$\frac{\left(-\frac{H_{1i}}{H_i}+K_{3i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}+K_{4i}=\Gamma_i^{-1} \qquad \text{(IV–27)}$$

$$\frac{\left(+\frac{H_{2i}}{H_i}+K_{3i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}+K_{4i}=\frac{U_{2i}}{\Gamma_i U_{1i}} \qquad \text{(IV–28)}$$

Hence

$$K_{4i}=\Gamma_i^{-1}-\frac{\left(-\frac{H_{1i}}{H_i}+K_{3i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)} \qquad \text{(IV–29)}$$

K_{3i} is determined by the following equation.

$$\left(-\frac{H_{1i}}{H_i}+K_{3i}\right)^{(1/n+1)}-\left(\frac{H_{2i}}{H_i}+K_{3i}\right)^{(1/n+1)}=\left(\frac{1}{n}+1\right)\left(\Gamma_i^{-1}-\frac{U_{2i}}{\Gamma_i U_{1i}}\right) \qquad \text{(IV–30)}$$

The volumetric flow rate Q can be expressed as

$$\frac{Q}{W}=\int_{-H_{2i}}^{+H_{1i}} v_{yi}\,dy=\Gamma_i U_{1i} H_i\int_{-H_{2i}/H_i}^{+H_{1i}/H_i}\phi_i\,d\bar{y}$$

$$=\Gamma_i U_{1i} H_i\left\{\frac{\left(-\frac{H_{1i}}{H_i}+K_{3i}\right)^{(1/n+2)}+\left(-\frac{H_{2i}}{H_i}+K_{3i}\right)^{(1/n+2)}}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)}+\frac{1}{\Gamma_i}-\frac{\left(+\frac{H_{1i}}{H_i}+K_{3i}\right)^{(1/n+1)}}{\left(\frac{1}{n}+1\right)}\right\} \qquad \text{(IV–31)}$$

Γ_i and K_{3i} are a function of $(p_i-p_{i-1})/S$.

(d) Figure III–2 (d), $(p_i-p_{i-1})/S<0$.

The velocity profile for (d) has an extreme value somewhere between two rolls, at $\bar{y}=\bar{y}^*_{di}$. In a similar way to (b), hence, it is necessary to write the differential equation for each region separately and account for the proper sign for the shear rate in the absolute value sign.

$$-\frac{d}{d\bar{y}}\left\{\left(-\frac{d\phi_{Ui}}{d\bar{y}}\right)^n\right\} = -1 \quad \left(+\frac{H_{1i}}{H_i} < \bar{y} < \bar{y}^*_{di}\right) \tag{IV-32}$$

$$\frac{d}{d\bar{y}}\left\{\left(\frac{d\phi_{Li}}{d\bar{y}}\right)^n\right\} = -1 \quad \left(\bar{y}^*_{di} < \bar{y} < -\frac{H_{2i}}{H_i}\right) \tag{IV-33}$$

The boundary conditions for (d) can be written as

$$\bar{y} = +\frac{H_{1i}}{H_i}: \quad \phi_{Ui} = \Gamma_i^{-1} \tag{IV-34}$$

$$\bar{y} = \bar{y}^*_{di}: \quad \frac{d\phi_{Ui}}{d\bar{y}} = 0 \tag{IV-35}$$

$$\bar{y} = \bar{y}^*_{di}: \quad \frac{d\phi_{Li}}{d\bar{y}} = 0 \tag{IV-36}$$

$$\bar{y} = -\frac{H_{2i}}{H_i}: \quad \phi_{Li} = \frac{U_{2i}}{\Gamma_i U_{1i}} \tag{IV-37}$$

$$\bar{y} = \bar{y}^*_{di}: \quad \phi_{Ui} = \phi_{Li} \tag{IV-38}$$

The solutions are

$$\phi_{Ui} = \frac{1}{\left(\frac{1}{n}+1\right)}\left\{\left(\frac{H_{1i}}{H_i}-\bar{y}^*_{di}\right)^{(1/n+1)} - (\bar{y}-\bar{y}^*_{di})^{(1/n+1)}\right\} + \Gamma_i^{-1} \tag{IV-39}$$

$$\phi_{Li} = \frac{-1}{\left(\frac{1}{n}+1\right)}\left\{(-\bar{y}+\bar{y}^*_{di})^{(1/n+1)} - \left(\frac{H_{2i}}{H_i}+\bar{y}^*_{di}\right)^{(1/n+1)}\right\} + \frac{U_{2i}}{\Gamma_i U_{1i}} \tag{IV-40}$$

where $\bar{y}^*_{bi}$ can be determined by the following equation obtained from the boundary condition (IV-38).

$$\left(\frac{1}{n}+1\right)\Gamma_i^{-1} - \left(\frac{H_{1i}}{H_i}-y^*_{di}\right)^{(1/n+1)} = \left(\frac{1}{n}+1\right)\frac{U_{2i}}{\Gamma_i U_{1i}} - \left(\frac{H_{2i}}{H_i}+\bar{y}^*_{di}\right)^{(1/n+1)} \tag{IV-41}$$

The volumetric flow rate Q is obtained by integrating the velocity equation.

$$\frac{Q}{W} = \Gamma_i U_{1i} H_i \left\{\int_{-H_{2i}/H_i}^{\bar{y}^*_{di}} \phi_{Li}\, d\bar{y} + \int_{\bar{y}^*_{di}}^{+H_{1i}/H_i} \phi_{Ui}\, d\bar{y}\right\} \tag{IV-42}$$

$$\int_{-H_{2i}/H_i}^{\bar{y}^*_{di}} \phi_{Li}\, d\bar{y} = \frac{1}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)}\left(\frac{H_{1i}}{H_i}+\bar{y}^*_{di}\right)^{(1/n+2)}$$

$$+\left\{\frac{1}{\left(\frac{1}{n}+1\right)}\left(\frac{H_{1i}}{H_i}+y^*_{di}\right)^{(1/n+1)} - \frac{U_{2i}}{\Gamma_i U_{1i}}\right\}\left(\bar{y}^*_{bi}+\frac{H_{2i}}{H_i}\right) \tag{IV-43}$$

$$\int_{\bar{y}_{di}^*}^{+H_{1i}/H_i} \phi_{Ui}\,d\bar{y} = \frac{-1}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)}\left(\frac{H_{1i}}{H_i}-\bar{y}_{di}^*\right)^{(1/n+2)} + \left\{\frac{1}{\left(\frac{1}{n}+1\right)}\left(\frac{H_{1i}}{H_i}-\bar{y}_{di}^*\right)^{(1/n+1)}+\Gamma_i^{-1}\right\}\left(\frac{H_{2i}}{H_i}-\bar{y}_{di}^*\right) \quad \text{(IV–44)}$$

Γ_i and $\bar{y}_{di}^*$ are a function of $(p_i-p_{i-1})/S$.

For the case where the geometrical configurations and the rotating velocities of a pair of rolls and further the flow behaviors of polymeric materials are known, the equation on the volumetric flow rate Q of (IV–9), (IV–20), (IV–31) or (IV–42), has N unknown p_i. Because the volumetric flow rate through each broken section is always equal to the constant Q and the both entrance and exit pressures in calendering are zero, the simultaneous equation of (IV–9), (IV–20), (IV–31) or (IV–42) contains N unknown p_i and hence the calendering problem is completely formulated.

Determination of the position of maximum pressure in calendering

In general there is a maximum pressure in calendering between entrance and nip. The position of maximum pressure at which the sign of pressure gradient $(p_i-p_{i-1})/S$ is varied, is very important also in the broken section analysis of calendering.

There is no pressure flow in the section of maximum pressure and hence the flow through this section is caused only by the drag flows driven by a pair of rolls. Therefore

$$\frac{Q}{W} = H_m \frac{(U_{1m}+U_{2m})}{2} \quad \text{(IV–45)}$$

where

H_m: height of the section of maximum pressure

U_{1m}: x-component of the peripheral velocity U_1 of upper roll at the position of maximum pressure

U_{2m}: x-component of the peripheral velocity U_2 of lower roll at the position of maximum pressure.

As shown in Figure II–1, H_m, U_{1m} and U_{2m} can be expressed in terms of α_{1m} and α_{2m}. Since α_{1m} has the geometrical relation to α_{2m}, the position of maximum pressure, i.e. α_{1m} or α_{2m}, can be determined by Eq. (IV–45).

V. DISCUSSION

The diameters of rolls are usually very large in comparison with the thickness of sheet or film of polymeric materials. Except the massive parts at the entrance region under multiple stress state, calendering problem with better accuracy in narrow nip region, can successfully be analyzed easily by broken section as well as lubrication theory.

In reality the pressure to the direction of roll axis is not uniform with a maximum at the mid surface of width. Together with rigorous analysis on the massive parts near the entrance region under complex flow field, the study on calendering should be developed three-dimensionally in the future.

The broken section method must also consider the spreading in the direction of the roll axis due to the pressure flow of the materials and also the normal stress[9] effect of polymeric materials.

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